647510 **MSC2MatC202x** **Seat No :_________**

M.Sc. (Maths) Semester - 2 (*CBCS*) Examination
May/June-2022 (NEW COURSE)
Complex Analysis - 2(CORE)

Time: 2:30 Hours **Marks: 70**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Q.1 (a) Attempt any TWO out of THREE. [10]

(1) Show that the composition of two bilinear transformations is bilinear.

(2) If $x_n = cis\frac{\pi}{2^n}$, then show that $x_1 . x_2 . x_3 \dots\ x_\infty = -1$.

(3) Explain conformal mapping and write properties of conformal mapping.

Q.1 (b) Attempt any ONE out of TWO. [4]

(1) Find a bilinear transformation that maps the region $D_1 : |z| > 1$ onto the region $D_2 : Re\, w < 0$.

(2) Write principal branches of $\log z$ and $z^{1/n}$.

Q.2 (a) Attempt any TWO out of THREE. [10]

(1) Suppose C is a simple closed curve around 0. Show that $\int_C \frac{1}{z} dz = 2\pi i$.

(2) Find a representation for the function $f(z) = \frac{1}{1+z}$ in negative powers of z.

(3) Find the Taylor series of $sin(z)$ around $z = 0$.

Q.2 (b) Attempt any ONE out of TWO. [4]

(1) State all the versions of Cauchy's theorem.

(2) Expand $f(z) = z^8 e^{3z}$ in a Taylor series around z = 0.

Q.3 (a) Attempt any TWO out of THREE. [10]

(1) State and prove fundamental theorem of algebra.

(2) Evaluate $\int_{-\infty}^{\infty} \frac{1}{(x^2+1)^2} dx$.

(3) Prove: Every entire and bounded function is constant.

Q.3 (b) Attempt any ONE out of TWO. [4]

(1) State maximum modulus theorem and minimum modulus theorem.

(2) State identity theorem.

Q.4 (a) Attempt any TWO out of THREE. [10]

(1) State and prove Inverse function theorem.

(2) Find the number of zeros of $p(z) = z^6 + 9z^4 + z^3 + 2z + 4$ in the unit disk.

(3) State argument principle with necessary explanations.

Q.4 (b) Attempt any ONE out of TWO. [4]

(1) State Schwarz Lemma with necessary explanations.

(2) Suppose f is analytic on the closed unit disk, $f(0) = 0$, and $|f(z)| \leq |e^z|$ whenever $|z| = 1$. How big can $f\left(\frac{1+i}{2}\right)$ be?

Q.5 (a) Attempt any TWO out of THREE. [10]

(1) Compute the Laurent series for the function $f(z) = \frac{z}{z^2+1}$ around $z_0 = i$. Give the region where the function is valid. .

(2) State and prove Residue theorem.

(3) Show that $z = 0$ is a pole of order 3 and $z = -1$ is a zero of order 1 for $(z) = \frac{z+1}{z^3(z^2+1)}$.

Q.5 (b) Attempt any ONE out of TWO. [4]

(1) Find the zero's of $(z-1)^3$.

(2) Classify the singularities.

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